$$\delta \int n(r)\, ds = \int \underbrace{n(f)\sqrt{f'^2(\varphi)+f^2(\varphi)}}_{L(f,f')}\, d\varphi$$

$$ds^2 = dr^2 + r^2 d\varphi^2$$

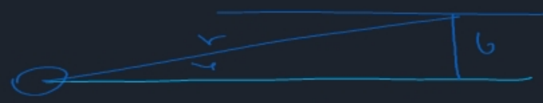
$$r = f(\varphi)$$

$$\frac{\partial L}{\partial f'} f' - L = \text{const}$$

$$\frac{\partial L}{\partial f'} = n \frac{f'}{\sqrt{f'^2+f^2}}$$

$$\frac{n(r)\, r^2}{\sqrt{r^2+r'^2}} = \text{const} = K$$

$$f' n \frac{f'}{\sqrt{f^2+f'^2}} - n\sqrt{f^2+f'^2} = \frac{n\left(f'^2-(f^2+f'^2)\right)}{\sqrt{}} = \frac{-n f^2}{\sqrt{f^2+f'^2}}$$



$$r \sin\varphi = b$$

$$r(\varphi) = \frac{b}{\sin\varphi}$$

$$r' = -\frac{b}{\sin^2\varphi}\cos\varphi$$

$$K = \frac{n(\infty)\, r^2}{\sqrt{}} = \frac{n(\infty)\, r^2}{r^2/b} = \underset{1}{n(\infty)}\, b = b$$

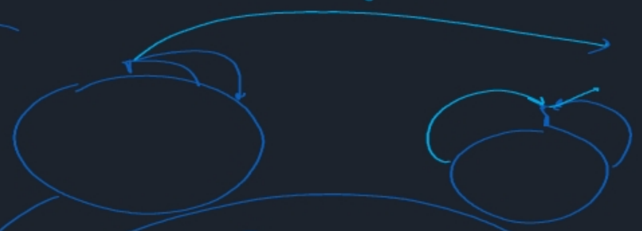
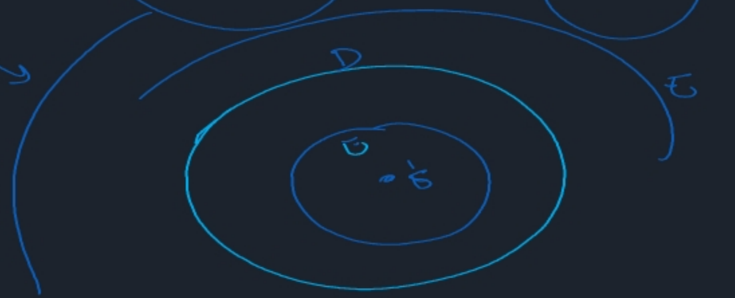
$$r'(P) = 0$$

$$K = n(P)\, R = b$$

$$\frac{b}{R} = n(R)$$

$$r'^2 + r^2 = \frac{b^2\cos^2\varphi}{\sin^4\varphi} + \frac{b^2}{\sin^2\varphi} = \frac{b^2}{\sin^4\varphi} = \frac{1}{b^2}\frac{b^4}{\sin^4\varphi} = \frac{r^4}{b^2}$$

$$\sqrt{r'^2+r^2} = \frac{r^2}{b}$$



$$\frac{n'(R)\, R}{n(R)} > 1$$